11. 02. 2026

* **Isolated point** A point $a \in A$ is said to be an isolated point of A iff it is not a limit point of A.

i.e.

iff $\exists$ a nbd N of a which contains no points of A other than a,

i.e.

iff $A \cap (N - \{a\}) = \phi$

* **Isolated set (or Discrete set)**

If every point of a set A is an isolated point of A $\Rightarrow$ A is called an isolated set.

* Let (X, d) be a metric space. Let $A \subset X$ and $B \subset X$. Let $x \in A$, $y \in B$.

Clearly $d(x, y) \geq 0$

$\therefore \{d(x,y) : x \in A, y \in B\}$ is a set of real numbers which is bounded below by zero and consequently this set must have a greatest lower bound or infimum.

* <u>Distance between two non-empty subsets A and B of (X, d)</u>

i.e. $d(A, B) = \text{g.l.b.}\{d(x, y) : x \in A, y \in B\}$
$= \text{Inf}\{d(x, y) : x \in A, y \in B\}$

If $A \cap B \neq \phi$ i.e. A and B have some common points then $d(x, y) = 0$ for these points.

$\therefore d(A, B) = \text{Inf}\{d(x, y) : x \in A\ y \in B\} = 0$

$\Rightarrow d(A, B) = 0$ if $A \cap B \neq \phi$

But If $A \cap B \neq \phi$ then it is not necessary that $d(A, B) = 0$. It is evident from the following example.

Let d be the usual metric on R i.e.

$d(x, y) = |x - y|$. Let $A = [0, 1[$, $B =]1, 2]$

i.e $A = \{x \in R : 0 \leq x < 1\}$ right hand open interval

and $B = \{x \in R : 1 < x \leq 2\}$ left open interval

$\Rightarrow A \cap B = \phi$ but $d(A, B) = 0$.